Written Exam for the M.Sc. in Economics 2010-I

# Contract Theory

Final Exam/Master’s Course

January 13, 2010

(4-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

**Attempt all three questions**

## Question 1

a) Explain what is meant by "bunching" (also called "pooling") in an adverse selection model. Argue with the help of a graphical analysis (and under the simplifying assumption that the agent's indifference curves are linear) that bunching is never optimal in a standard two-type adverse selection model. Can bunching occur in a three-type adverse selection model (no proof or argument is required)?

b) Explain in words what the revelation principle is and why it is a useful result.

c) Explain briefly the design of the experiment that is reported in Anderhub, Gächter and Königstein (*Experimental Economics*, 2002). In the paper, the authors make a list of nine "observations" that summarize their experimental results. Give a brief account of five of these observations.

d) In the part of the course that was based on the paper by Vickers (*Oxford Economic Papers*, 1995) we studied a model with no explicit incentives but where the agent was disciplined by implicit incentives. Explain in words the logic of that model (the single-agent case suffices).

## Question 2 (adverse selection)

Consider the following model of a market for pencils that can be produced in different qualities. There are a continuum of consumers (the "agent" of the adverse selection model), each of whom purchasing either one pencil or no pencil. A fraction $\nu \in (0,1)$ of the consumers have a high valuation for pencil quality and the remaining fraction $(1-\nu)$ have a low valuation for pencil quality (and the total number of consumers is normalized to one). The high-valuation consumers' payoff if consuming one pencil of quality $\overline{q}$ at the price $\overline{t}$ is given by

$$\overline{\theta}\overline{q} - \overline{t},$$

where $\overline{\theta} > 0$ is a parameter. The low-valuation consumers' payoff if consuming one pencil of quality $\underline{q}$ at the price $\underline{t}$ is given by

$$\underline{\theta}\underline{q} - \underline{t},$$

where $\underline{\theta}$ is a parameter satisfying $\overline{\theta} > \underline{\theta} > 0$. If the consumers (both the high- and low-valuation ones) choose not to consume any pencil at all, their payoff is zero. There is a firm (the "principal" of the adverse selection model) that has a monopoly in the pencil market. If selling one pencil of quality $\underline{q}$ to each of the low-valuation consumers and one pencil of quality $\overline{q}$ to each of the high-valuation consumers, the firm incurs the production costs

$$\frac{1-\nu}{2}\underline{q}^2 + \frac{\nu}{2}\overline{q}^2.$$

The firm's total profits are therefore given by

$$(1-\nu)\,\underline{t} + \nu\bar{t} - \frac{1-\nu}{2}\underline{q}^2 - \frac{\nu}{2}\overline{q}^2.$$

Each consumer knows his or her own $\theta$ perfectly. However, the monopoly firm does not know the $\theta$ of an individual consumer, but only that a fraction $\nu$ of the consumers have a high valuation and that the rest have a low valuation. The objective of the firm is to maximize its total profits.

a) Suppose the parameters are such that the firm optimally interacts with both kinds of consumers. Formulate the optimization problem that the firm faces when designing the menu of prices: state the objective function and the constraints, and explain what the choice variables are. Explain the meaning of the constraints in words.

b) Prove formally that any pair of qualities $(\underline{q}, \overline{q})$ that satisfy the constraints under a) also satisfy $\underline{q} \leq \overline{q}$. Illustrate the argument of the proof in a diagram with $q$ and $t$ on the axes.

c) Let the first-best levels of $\underline{q}$ and $\overline{q}$ be defined as the ones that maximize the total surplus,

$$(1-\nu)\,\underline{\theta}\underline{q} + \nu\overline{\theta}\overline{q} - \frac{1-\nu}{2}\underline{q}^2 - \frac{\nu}{2}\overline{q}^2.$$

Calculate these first-best levels. Explain the economic intuition behind your result.

d) Now return to the second-best problem you have formulated under a). Solve this problem. Explain how the optimal second-best qualities differ from the optimal first-best qualities. Also explain the economic intuition behind any differences. Which type, if any, gets any rents at the second-best optimum? Why?

**Question 3 (moral hazard)**

Prometheus Sørensen (the principal, P for short) owns a factory producing pencils and wants to hire Absalon Nielsen (the agent, A for short) to work there. If hired, A's task will be to operate a pencil machine and to make sure it runs smoothly. To do this well, A must "make an effort", which involves a (personal) cost to A. This is modelled as A's choosing an effort level $e \in \{0,1\}$, where $e = 1$ means "making an effort" and $e = 0$ means "not making an effort". The associated cost equals

$$\psi(e) = \begin{cases} \psi & \text{if } e = 1 \\ 0 & \text{if } e = 0, \end{cases}$$

with $\psi > 0$. The number of pencils that come out of the machine, $q$, is either large ($q = \overline{q}$) or small ($q = \underline{q}$), with $\overline{q} > \underline{q} > 0$. The probability that the number is large depends on whether A has made an effort or not:

$$\Pr(q = \overline{q} \mid e) = \begin{cases} \pi_1 & \text{if } e = 1 \\ \pi_0 & \text{if } e = 0, \end{cases}$$

with $0 < \pi_0 < \pi_1 < 1$. P (and the court) can observe which quantity that is realized ($\overline{q}$ or $\underline{q}$) but not the effort level chosen by A. It is assumed that P has all the bargaining power and makes a take-it-or-leave-it offer to A. A contract can specify two numbers, $\bar{t}$ and $\underline{t}$, where $\bar{t}$ is the payment to A if $q = \overline{q}$, and $\underline{t}$ is the payment to A if $q = \underline{q}$. P is risk neutral and his payoff, given a quantity $q$ and a payment $t$, equals

$$V = q - t.$$

A is also risk neutral and his payoff, given a payment $t$ and an effort level $e$, equals

$$U = t - \psi(e).$$

A is protected by limited liability, meaning that $\bar{t} \geq 0$ and $\underline{t} \geq 0$. A's outside option would yield the payoff $\widehat{U} \geq 0$.

a) Assume that $\widehat{U} = 0$. Calculate (analytically, not using a figure) P's cost of implementing the high effort level when (i) P can observe A's effort (i.e., the first best) and (ii) when P cannot observe A's effort (i.e., the second best). Compare these costs and explain in what sense effort is underprovided in the model due to asymmetric information. How would the conclusion change if $\pi_0 = 0$? Explain the economic significance of the assumption that $\pi_0 > 0$.

b) Relax the assumption that $\widehat{U} = 0$ and allow for any $\widehat{U} \geq 0$. Only consider the case where P wants to induce A to make an effort. Illustrate the second-best solution in a diagram with $\bar{t}$ on the vertical axis and $\underline{t}$ on the horizontal axis. Show in the figure and explain, in qualitative terms, how the nature of the second best solution changes as the outside option utility $\widehat{U}$ becomes larger.

c) Suppose that the agent is not protected by limited liability. Explain in words how and why this affects the nature of the second-best solution.